Fuzzy Logic Inference of Dark Matter Mass through Gravitational Lensing

A graph with a rainbow colored curve

Description automatically generated with medium confidence

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Contents

[Dark Matter Classification – Project Proposal 3](#_Toc164365326)

[Title: Fuzzy Logic Inference of Dark Matter Mass through Gravitational Lensing 4](#_Toc164365327)

[Introduction 4](#_Toc164365328)

[Fuzzy Logic & Dark Matter Beginnings 4](#_Toc164365329)

[Research: 5](#_Toc164365330)

[Implementation 6](#_Toc164365331)

[Run 1 Notes – Centroid 7](#_Toc164365332)

[PCA Variance Explained: 7](#_Toc164365333)

[Rule Viewer Plot: 7](#_Toc164365334)

[Fuzzy Inference System Output - Surf Plot: 7](#_Toc164365335)

[Sensitivity Analysis for Angular Resolution Plot: 7](#_Toc164365336)

[Conclusion and Next Steps: 8](#_Toc164365337)

[Adjustments Made: 8](#_Toc164365338)

[Notes for further testing: 10](#_Toc164365339)

[Further Research Notes 11](#_Toc164365340)

[Reference list 12](#_Toc164365341)

[Appendix 12](#_Toc164365342)

[System Creation & Plan Appendices 12](#_Toc164365343)

[1. Define the Fuzzy System Objectives 12](#_Toc164365344)

[Fuzzy Logic Model Development: 12](#_Toc164365345)

[2. Identify Input and Output Variables 13](#_Toc164365346)

[3. Construct Membership Functions 14](#_Toc164365347)

[4. Formulate the Rule Base 14](#_Toc164365348)

[5. Implement Inference Engine 16](#_Toc164365349)

[6. Defuzzification 16](#_Toc164365350)

[Data 16](#_Toc164365351)

[Testing Appendices 18](#_Toc164365352)

[Mathematical Formulas Appendices 18](#_Toc164365353)

[Code Appendices 18](#_Toc164365354)

[Type 1 Mamdani Code 18](#_Toc164365355)

[Adapting Angular Resolution to Fuzzy Models 22](#_Toc164365356)

[Wrap-around Membership Functions 22](#_Toc164365357)

[Vector-based Approach 22](#_Toc164365358)

[Multiple Angles Input 23](#_Toc164365359)

[Domain-specific Transformation 23](#_Toc164365360)

# Dark Matter Classification – Project Proposal

I came across a very interesting article which spurred my idea for this system. Here is the link: <https://www.mpa-garching.mpg.de/1076681/hl202306>

Embarking on a project that intersects the captivating realms of astronomy and artificial intelligence presents an exhilarating opportunity to explore the enigmatic universe in which we reside. My encounter with an insightful article from the Max Planck Institute for Astrophysics, "The Mystery of Dark Matter," has ignited a fervent curiosity within me to delve deeper into this celestial puzzle. Herein lies the foundation of my proposed research project.

Astronomy has always been a subject that invites wonder and inquiry, and my aspiration is to enhance my research skills within this vast field. The phenomenon of dark matter, which binds the galaxies together yet eludes direct detection, stands as one of modern science's most alluring challenges. By investigating this shadowy component of the cosmos, I intend not only to satisfy personal intellectual curiosity but also to contribute to our collective understanding of the universe's fundamental structure.

In pursuit of this aim, I propose to develop a fuzzy inference system that can estimate the mass of dark matter based on astrophysical observations. This system will employ a combination of angular resolution, density, and width of gravitational arcs as input parameters, reflecting the interactions of dark matter with observable phenomena. Although my current stage as a budding researcher might not lead to groundbreaking discoveries, this endeavour is a significant stepping stone towards a deeper engagement with astrophysics.

The desired outcome of this project is twofold. Academically, it is to design and implement a sophisticated computational model that can offer reliable predictions about dark matter's mass—a feature crucial to our understanding of galactic formations and behaviour. On a personal level, the project aims to develop a robust framework for scientific inquiry and data analysis within astrophysics, paving the way for future research endeavours that could potentially unveil novel insights into the dark universe.

In essence, this research project is driven by a profound passion for astronomy and a resolute commitment to advancing the frontiers of knowledge. The fusion of artificial intelligence with celestial studies represents a promising avenue to address some of the most profound questions that have captivated humanity since we first gazed upon the night sky in wonder. Through this project, I aspire to contribute to the grand tapestry of cosmic exploration and discovery.

# Title: Fuzzy Logic Inference of Dark Matter Mass through Gravitational Lensing

# Introduction

The elusive nature of dark matter presents one of the most captivating puzzles in contemporary astrophysics. Dark matter's reluctance to interact via electromagnetic forces—yet its gravitational influence on cosmic structures—is a phenomenon that necessitates innovative approaches to detection and analysis. The study at hand utilises a type 1 Fuzzy Logic System (FLS) to model and infer the mass of dark matter particles, harnessing the mathematical framework that accommodates the inherent uncertainties of astrophysical observations.

Fuzzy logic, with its origins rooted in the concept of partial truth, offers a compelling framework for modelling the indeterminate and complex phenomena characteristic of astrophysical systems. Unlike binary logic, which rigidly dichotomises truth values, fuzzy logic introduces a spectrum of truth—a feature that resonates profoundly with the probabilistic and observational nature of cosmological research. This project specifically addresses the challenge of determining the mass of dark matter particles by analysing data from gravitational lensing events. Gravitational lensing—the bending of light from distant celestial bodies by the gravitational field of a massive object such as a galaxy—provides indirect but powerful evidence of dark matter's presence and properties.

By employing a fuzzy logic approach, it is possible to construct a model that interprets the "fuzziness" or uncertainty in the gravitational lensing data—such as the angular resolution of the observation, the density of the gravitational lens, and the width and distribution of gravitational arcs. This model not only encapsulates the non-linear and multifactorial relationships between these variables but also yields an estimation of the dark matter particle mass.

The proposed Fuzzy Inference System (FIS) is predicated on the principle that astrophysical measurements are frequently accompanied by ambiguity and imprecision. Traditional methods of data analysis may fall short in capturing the subtleties and gradations of such data. In contrast, the FLS accommodates these nuances, providing a robust and versatile tool for inference. The system's rules encapsulate the expert understanding of the interplay between the observed variables and the mass of dark matter particles, translating this knowledge into a series of mathematical relationships. The output—a quantified estimate of the mass—thus emerges from a holistic appraisal of multiple, interacting factors.

This project's ultimate aim is to contribute a novel methodological perspective to dark matter research. By leveraging the strengths of fuzzy logic, it seeks to enhance our ability to interpret complex astrophysical data and enrich our comprehension of the universe's unseen matter. The endeavour represents a confluence of astrophysics, applied mathematics, and computer science, illustrating the interdisciplinary synergy essential for advancing our cosmic frontier.

## Fuzzy Logic & Dark Matter Beginnings

Fuzzy logic, a term and concept conceived by Lotfi A. Zadeh, revolutionised the way we handle imprecision and reasoning in computing. Introduced in his seminal 1965 paper, fuzzy logic provided a structured, logical way to deal with the ambiguous and imprecise nature of human language and thought. Unlike traditional binary logic systems that are definitive and rigid, fuzzy logic allows for degrees of truth rather than a simple true or false dichotomy. This approach has become foundational in the development of artificial intelligence and control systems, allowing for more nuanced decision-making that better mirrors human reasoning​.

Dark matter, on the other hand, has been a subject of intrigue since its effects were first inferred by Fritz Zwicky in the 1930s through the observation of the Coma Cluster, (Zwicky, 1937). The quest to comprehend dark matter has unfolded over decades, marking a pivotal chapter in cosmology. As charted by the seminal 'History of Dark Matter' review in Reviews of Modern Physics, (Bertone and Hooper, 2018), this journey spans from the nascent observational clues to the robust theoretical frameworks that now underscore its existence (Reference: Reviews of Modern Physics). This historical context enriches the foundation upon which the current study is built, underscoring the enduring quest to demystify the cosmos's unseen constituents. The term itself encapsulates the unknowns—matter that does not emit, absorb, or reflect light, making it invisible and detectable only through its gravitational effects. It's a cornerstone of contemporary cosmology, representing a fundamental aspect of the universe that remains enigmatic. While the concept has been around for decades, its exact nature is still one of the most compelling mysteries in physics and astronomy. (de Swart, 2019).

Bringing together fuzzy logic with the exploration of dark matter introduces a unique interdisciplinary challenge. Such an endeavour combines the abstract mathematical frameworks of fuzzy systems with the empirical observations of cosmology. The potential to apply fuzzy logic to dark matter research holds promise for new insights, possibly allowing the handling of uncertainties and complexities inherent in cosmic phenomena. This represents an exploratory leap that could expand our methods of investigating and understanding the universe.

# Research:

In 2023, the Max Planck Institute published a significant article detailing the latest findings in the field of dark matter research, focusing on the theoretical model known as Fuzzy Dark Matter (FDM). The study sheds light on the elusive nature of dark matter, which constitutes a substantial portion of the universe's mass yet remains undetectable through electromagnetic means, revealing itself only through gravitational interactions. The article emphasises the potential of FDM to resolve longstanding puzzles in astrophysics, such as the cusp-core problem and the missing satellite problem, which challenge traditional Cold Dark Matter (CDM) models. By proposing dark matter particles with extremely low masses, the FDM model, characterised by ultra-light particles with masses around 10−22 eV, influencing the distribution and behavior of dark matter in galaxies. Key experiments and observations highlighted in the article, including gravitational lensing and cosmic microwave background analyses, offer insights into how FDM could alter our understanding of cosmic structure formation. The research conducted by the Max Planck Institute demonstrates the interdisciplinary approach required to probe the nature of dark matter, combining theoretical physics, astrophysics, and advanced observational techniques.

The FDM model was not introduced by a single, definitive paper; rather, it gained traction through a series of studies exploring the implications of ultra-light scalar fields as dark matter candidates. Among the influential early works in this area are; Hu, Barkana, and Gruzinov, (2000), who discussed the core properties of ultralight scalar particles as dark matter candidates. They highlighted how such particles could address issues like the cusp-core problem in the density profiles of dwarf galaxies. Marsh and Ferreira’s paper, (2010), further explored the implications of ultra-light scalar fields on cosmic structure formation and provided constraints on the mass of these particles based on cosmological observations.

These papers, among others, laid the foundational theoretical framework for FDM, providing insights into how ultra-light particles could behave as dark matter and affect the formation and evolution of cosmic structures. The field has since grown, with numerous studies using observations from galaxy rotation curves, gravitational lensing, and cosmic microwave background measurements to constrain the properties of FDM and differentiate it from other dark matter models.

# Implementation

In the development of the FIS for the task of estimating the mass of dark matter based on astronomical observations, several key components were selected to construct a comprehensive model that could provide meaningful and interpretable results.

The Mamdani-type FIS was chosen due to its intuitive appeal and the interpretability of its fuzzy rules, which are akin to human reasoning. This type of system is well-suited for problems where the relationships between variables are complex and not easily defined by traditional quantitative analysis. By utilising fuzzy logic, the system can handle the inherent uncertainty and imprecision associated with the measurements of astronomical phenomena.

The input variables — angular resolution, density of the gravitational lens, and width and distribution of gravitational arcs — were identified based on their significance in the context of gravitational lensing. Angular resolution is critical as it determines the system's ability to discern fine details in the observed lensing patterns. The density variable quantifies the lensing mass's influence on the light distortion, thus offering insight into the mass distribution within the lens galaxy. The width and distribution of arcs provide spatial information about the lensing effect, which is pivotal for understanding the structure and extent of dark matter.

The range of each input variable was carefully defined to encompass all possible values observable in this astronomical context. Membership functions for each input were designed to capture the low, medium, and high gradations, with trapezoidal membership functions selected for the boundary values to accommodate the fuzziness at the extremes of the ranges and triangular membership functions for the intermediate values to represent the gradual transition between states.

The output variable — the estimated mass of dark matter — was similarly defined with membership functions representing light, medium, and heavy masses. The granularity of these functions was determined to offer a nuanced view of the mass estimation, which is crucial for subsequent astrophysical analysis and interpretation.

A comprehensive rule base was formulated, capturing the expert understanding of the interactions between the inputs and the resultant mass estimation. The rules were constructed to reflect the varying degrees of influence that different observational parameters have on the perceived mass. These rules are pivotal in translating the fuzzy inputs into a fuzzy output, which is then defuzzified to produce a quantifiable estimate.

In terms of defuzzification methods, the Centroid method was primarily used for its balance and mathematical robustness, providing a weighted average that is often seen as the most representative value when considering the shape of the fuzzy set. However, to fully explore the model's behaviour and sensitivity to different defuzzification strategies, additional methods such as Mean of Maximum (MoM), Largest of Maximum (LoM), Smallest of Maximum (SoM), and Bisector were also evaluated. This multifaceted approach allowed for a comparative analysis to discern which method aligned best with the empirical data and theoretical expectations.

The constructed FIS is a thoughtful amalgamation of fuzzy logic principles applied to a domain of significant scientific intrigue. Each element of the system was chosen and calibrated to contribute towards a nuanced understanding of the relationship between observable gravitational effects and the elusive dark matter, which continues to be a cornerstone challenge in modern astrophysics.

# Change of Input Variables

In response to the complexities inherent in modelling the curvature of angular resolution within a Type 1 FIS, a strategic adjustment was made to the selection of input variables. The focus shifted towards measurements that are more amenable to linear representation yet still crucial in the study of dark matter through astrophysical phenomena. Specifically, galaxy rotation curves, gravitational lensing effects, and cosmic microwave background measurements were chosen. These variables not only align with the original criteria of significance in gravitational lensing but also offer robust, quantifiable data that can be effectively handled by the FIS without necessitating the complex adaptations required for angular resolution's circular nature. This realignment ensures the FIS remains practical while still capturing essential aspects of cosmic phenomena related to dark matter.

Galaxy Rotation Curves: These are graphs that show the variation of the orbital velocities of stars or gas in galaxies as a function of their distance from the galaxy’s centre. The observed flatness of the rotation curves at distances where the influence of visible matter is negligible suggests the presence of dark matter.

Gravitational Lensing: This phenomenon occurs when the gravity of a massive object, like a cluster of galaxies (acting as a lens), warps the space around it and bends the path of light from a background object. Analysis of the lensing effect can reveal the mass distribution of the lens, including any dark matter.

Cosmic Microwave Background (CMB) Measurements: The CMB is the thermal radiation left over from the time of recombination in Big Bang cosmology. Measurements of the CMB’s temperature fluctuations provide a snapshot of the universe at a young age, allowing for the indirect measurement of dark matter through its gravitational effects on the early universe's density fluctuations.

## Justifications

### Justification for the Rulebase

The rulebase for the FIS is meticulously designed to embody the interactions and dependencies among the key observational parameters of galaxy rotation, gravitational lensing effects, and CMB measurements. Each rule encapsulates how these variables collectively influence our understanding of dark matter mass. For instance, the combination of high galaxy rotation with pronounced gravitational lensing effects suggests greater mass, mirroring astrophysical expectations where increased rotational velocities and lensing are indicative of substantial dark matter presence.

### Justification for the Membership Functions

The chosen membership functions—trapezoidal and triangular—serve to reflect the continuity and ambiguity inherent in astrophysical data. Trapezoidal functions are utilised at the boundaries of input domains, allowing for a gradual transition from full membership to non-membership, which is crucial given the uncertain and extremal nature of observational data. Triangular functions, applied to the central regions of the input domains, facilitate a clear delineation of the states between minimal and maximal observations, ensuring that the system can capture the subtle gradations in the data effectively. This configuration not only supports robustness in modelling but also enhances the interpretability and responsiveness of the system to varying astrophysical conditions.

### Justification for Visualisations

These visualisations are chosen to offer a comprehensive overview of the Fuzzy Inference System (FIS) at various stages of the model. The first set visualises the membership functions for the inputs and the output, providing an immediate graphical representation of how data is categorised within the system. The surface plot is employed to illustrate the interaction effects between multiple inputs on the predicted output, essential for understanding complex relationships within the data. The rule viewer assists in validating the logical structure of the fuzzy rules, ensuring they perform as expected. Sensitivity analysis is crucial for identifying which inputs have the most significant impact on outputs, helping prioritise focus areas in further research and adjustments. Lastly, the PCA is used to identify the principal components, offering insights into the underlying structure of the data, which is vital for dimensionality reduction and feature importance analysis.

## Using Python for pre-analysis and data generation

Prior to employing the fuzzy system for data analysis, a preliminary analysis in Python involves generating and normalising data for the new input variables: galaxy rotation curves, cosmic microwave background, and gravitational lensing measurements. This pre-analysis stage includes simulating realistic datasets based on astrophysical principles and normal distributions, where each input variable is transformed to ensure a uniform scale suitable for fuzzy processing. This standardisation is critical as it ensures that the fuzzy system evaluates each input equitably, enhancing the model's accuracy and reliability in estimating dark matter mass.

## First Run With The New Variables

Test 1 of the FIS, integrating new astronomical variables—galaxy rotation curves, gravitational lensing effects, and cosmic microwave background measurements—yielded insightful initial outcomes. The membership functions (Fig. 1) were adeptly crafted, capturing the nuances across the spectrum of each variable's domain, yet a few rules within the system did not fire as anticipated, signifying potential refinements in overlap and coverage. The rule viewer visualization (Fig. 2) provided a detailed illustration of the system's operational logic under median input conditions, indicating a robust interaction between inputs and the estimated dark matter mass.

Sensitivity analysis (Figs. 3, 4, and 5) displayed the system's responsiveness to variations in individual variables, revealing which influenced the output most significantly. Surface plots (Figs. 6 and 7) further explicated the intricate interplay between pairs of input variables. Principal Component Analysis (Fig. 8) underscored the dominance of certain variables, offering a pathway to reduce dimensionality without substantial information loss.

Consequently, the next phase of testing will focus on optimising membership function overlap and broadening the rule base to ensure a comprehensive response spectrum across all inputs. This phase will also investigate alternative defuzzification strategies, aiming to refine the granularity of output and enhance the interpretive power of the FIS for dark matter mass estimation.

## Second Run With Increased Membership Across Input Variables

# Further Research Notes

Given that the problem of detecting and analysing dark matter is inherently uncertain and complex, with a lot of unknowns and noise in the data, a type-2 fuzzy system could potentially yield better results. It could provide a more nuanced handling of uncertainties which are characteristic of dark matter observations. This would align with the goal of achieving an excellent system. Further, in the research and development, it would be good to incorporate an analysis of the uncertainty levels in the input data and how these could affect the conclusions drawn from the system. This might involve comparing the outcomes of type-1 and type-2 systems using the same datasets to observe any improvements in robustness and accuracy with the type-2 approach. If time allows this will be something to implement.

## The Challenge of Circularity in Fuzzy Inference Systems

Inherently, fuzzy systems don’t do well with data that is circular, for example, degrees or clock/time. The challenge of incorporating angular resolution as an input in fuzzy logic systems stems from its intrinsic circular nature, posing a periodic boundary condition often ill-suited to linear membership functions. To circumvent this, implementing wrap-around membership functions that acknowledge the cyclical continuity. Alternatively, a vector-based representation, decomposing the angle into its sine and cosine constituents, could preserve the proximity of adjacent angular values. A dual angle input, leveraging the trigonometric transformation of angles into a bi-dimensional plane, also offers a solution, capturing the essence of circularity. These methods could render the variable more amenable to fuzzy categorisation. This reframing of the input could elucidate otherwise obscure relationships within the data, offering a more robust basis for the inference system. Each one will be attempted for analysis, and a decision of the best route forward can be taken with confidence.

|  |  |
| --- | --- |
| Wrap-around Membership Functions: | Design membership functions that wrap around the endpoints to effectively model the circular nature. |
| Vector-based Approach: | Represent the angles as vectors and use their components as inputs to your FIS. This way, the circular nature is inherently captured since angles that are close to each other will have similar vector representations. |
| Multiple Angles Input: | Sometimes, using cosines and sines of angles as separate inputs can help to capture the circular nature in a two-dimensional space. |
| Domain-specific Transformation: | Transform the angular resolution into another metric that represents the quality or clarity which may be linear. |

Table.. – Possible approaches to combat potential issues with the circular nature of Angular Resolution

# Reference list

Hu, W., Barkana, R. and Gruzinov, A. (2000). Fuzzy Cold Dark Matter: The Wave Properties of Ultralight Particles. *Physical Review Letters*, 85(6), pp.1158–1161. doi:<https://doi.org/10.1103/physrevlett.85.1158>.

Marsh, D.C. and Pedro Lopes Ferreira (2010). Ultra-Light Scalar Fields and the Growth of Structure in the Universe. *arXiv (Cornell University)*, 1(1). doi:<https://doi.org/10.1103/physrevd.82.103528>.

Max Planck Institute (2023). *If Dark Matter Is fuzzy, Then How Fuzzy Is it? - a Gravitational Lens Has the Answer*. [online] www.mpa-garching.mpg.de. Available at: <https://www.mpa-garching.mpg.de/1076681/hl202306> [Accessed 12 Apr. 2024].

Sánchez-Lozano, J.M., Moya, A. and Rodríguez-Mozos, J.M. (2021). A fuzzy Multi-Criteria Decision Making approach for Exo-Planetary Habitability. *Astronomy and Computing*, 36(1), p.100471. doi:<https://doi.org/10.1016/j.ascom.2021.100471>.

# Appendix

## Plain English Rulebase Descriptions:

If galaxy rotation is slow, lensing effects are weak, and CMB is low, then mass is light.

If galaxy rotation is slow, lensing effects are weak, and CMB is moderate, then mass is light.

If galaxy rotation is slow, lensing effects are weak, and CMB is high, then mass is light.

If galaxy rotation is slow, lensing effects are moderate, and CMB is low, then mass is light.

If galaxy rotation is slow, lensing effects are moderate, and CMB is moderate, then mass is medium.

If galaxy rotation is slow, lensing effects are moderate, and CMB is high, then mass is light.

If galaxy rotation is slow, lensing effects are strong, and CMB is low, then mass is medium.

If galaxy rotation is slow, lensing effects are strong, and CMB is moderate, then mass is medium.

If galaxy rotation is slow, lensing effects are strong, and CMB is high, then mass is medium.

If galaxy rotation is moderate, lensing effects are weak, and CMB is low, then mass is light.

If galaxy rotation is moderate, lensing effects are weak, and CMB is moderate, then mass is light.

If galaxy rotation is moderate, lensing effects are weak, and CMB is high, then mass is light.

If galaxy rotation is moderate, lensing effects are moderate, and CMB is low, then mass is medium.

If galaxy rotation is moderate, lensing effects are moderate, and CMB is moderate, then mass is medium.

If galaxy rotation is moderate, lensing effects are moderate, and CMB is high, then mass is medium.

If galaxy rotation is moderate, lensing effects are strong, and CMB is low, then mass is heavy.

If galaxy rotation is moderate, lensing effects are strong, and CMB is moderate, then mass is medium.

If galaxy rotation is moderate, lensing effects are strong, and CMB is high, then mass is heavy.

If galaxy rotation is fast, lensing effects are weak, and CMB is low, then mass is medium.

If galaxy rotation is fast, lensing effects are weak, and CMB is moderate, then mass is light.

If galaxy rotation is fast, lensing effects are weak, and CMB is high, then mass is light.

If galaxy rotation is fast, lensing effects are moderate, and CMB is low, then mass is heavy.

If galaxy rotation is fast, lensing effects are moderate, and CMB is moderate, then mass is medium.

If galaxy rotation is fast, lensing effects are moderate, and CMB is high, then mass is medium.

If galaxy rotation is fast, lensing effects are strong, and CMB is low, then mass is heavy.

If galaxy rotation is fast, lensing effects are strong, and CMB is moderate, then mass is heavy.

If galaxy rotation is fast, lensing effects are strong, and CMB is high, then mass is heavy.

## Testing Appendices

### Previous Input Variables Visualisations

A diagram of a graph

Description automatically generated with medium confidenceA diagram of a light and heavy weight

Description automatically generated

A graph with blue rectangular bars

Description automatically generatedA screenshot of a graph

Description automatically generated

A graph with a line pointing at the top

Description automatically generated with medium confidenceA graph with a rainbow colored curve

Description automatically generated with medium confidence

Figure.. – First Testing Run Visualisations Prior to Changes Being Implemented (For reference of Journey)

### Test 1 Notes & Figures

NB: This run is post input variable changes.

A graph of a graph

Description automatically generated with medium confidenceA graph of rhombuses

Description automatically generated

Figure.. – Test One: Input & Output Membership Functions

A graph of a graph

Description automatically generatedA graph of a curve

Description automatically generated

Figure.. – Test One: Sensitivity Analysis of GLE & CMB

A graph of a function

Description automatically generated

Figure.. – Test One: Sensitivity Analysis of GRC

A screenshot of a computer

Description automatically generated

Figure.. – Test One: Rule Viewer – (NB: Not all rules fired)

A graph showing a rainbow colored graph

Description automatically generated with medium confidenceA graph of a function

Description automatically generated with medium confidence

Figure.. – Test One: Smoothed Surf Plot & Rule Viewer Surf Plot

## Mathematical Formulas Appendices

## Code Appendices

### EstimatingDarkMatterMass.m

### evaluateAndWriteFIS.m

### generateSurfPlot.m

function generateSurfPlot(fis, fixedWidthValue)

% Generate input grids for Galaxy Rotation Curve and Cosmic Microwave Background

rotationCurveRange = linspace(0, 1, 50); % Range for Galaxy Rotation Curve

cmbRange = linspace(0, 1, 50); % Range for Cosmic Microwave Background

[RotationCurve, CMB] = meshgrid(rotationCurveRange, cmbRange);

% Initialise an output matrix for the Dark Matter Mass results

Mass = zeros(size(RotationCurve));

% Evaluate the FIS for each combination of RotationCurve and CMB, with fixed gravitational lensing width

for i = 1:size(RotationCurve, 1)

for j = 1:size(RotationCurve, 2)

input = [RotationCurve(i,j), CMB(i,j), fixedWidthValue]; % ensure these are normalised

output = evalfis(fis, input);

if isempty(output) || isnan(output)

disp('No output for input: ');

disp(input);

Mass(i,j) = NaN; % Assign NaN to indicate no output

else

Mass(i,j) = output;

end

end

end

% Check the size of the Mass matrix

disp(['Size of Mass matrix: ', num2str(size(Mass))]);

% If Mass is not a matrix, display an error message

if isvector(Mass) || isscalar(Mass)

error('Mass must be a matrix. Check your FIS configuration and inputs.');

end

% Create the surface plot

figure; % Create a new figure

surf(RotationCurve, CMB, Mass); % Generate the surface plot

xlabel('Galaxy Rotation Curve');

ylabel('Cosmic Microwave Background');

zlabel('Estimated Dark Matter Mass (eV)');

title('Fuzzy Inference System Output: Estimated Dark Matter Mass');

shading interp; % Interpolate colors across surfaces and patches

colorbar; % Display colorbar

end